B-Meson Wave Function through A Comparative Analysis of the $B \to \pi$, K Transition Form Factors

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Abstract

The properties of the B-meson light-cone wave function up to next-to-leading order Fock state expansion have been studied through a comparative study of the $B \to \pi$, K transition form factors within the k_T factorization approach and the light-cone sum rule analysis. The transition form factors $F^{B\to\pi}_{+,0,T}$ and $F^{B\to K}_{+,0,T}$ are carefully re-calculated up to $\mathcal{O}(1/m_b^2)$ within the k_T factorization approach in the large recoil region, in which the main theoretical uncertainties are discussed. The QCD light-cone sum rule is applicable in the large and intermediate energy regions, and the QCD light-cone sum rule results in Ref.[19] are adopted for such a comparative study. It is found that when the two phenomenological parameters $\bar{\Lambda} \in [0.50, 0.55]$ and $\delta \in [0.25, 0.30]$, the results of $F^{B\to\pi}_{+,0,T}(Q^2)$ and $F^{B\to K}_{+,0,T}(Q^2)$ from these two approaches are consistent with each other in the large recoil energy region.

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The non-perturbative light-cone (LC) wavefunction (WF) of the B meson plays an important role for making reliable predictions for exclusive B meson decays. However, the B-meson WF still poses a major source of uncertainty in the study of B meson decays. Unless we have known it well and applied it for some precise studies, we can not definitely say that there is really new physics in the B-meson decays, e.g. the so called $B \to \pi K$ puzzle [1] and etc.. Hence, theoretically, it is an important issue to study on it.

An analytic solution for the leading Fock-state B-meson WF, which is derived under the Wandzura-Wilczek (WW) approximation [4] and with the help of the equation of motion of the light spectator quark in the B meson, has been given in Refs. [2, 3]. It shows that the leading Fock-state B-meson WF can be determined uniquely in analytic form in terms of the "effective mass" ($\bar{\Lambda}$) [5] of the meson state and its transverse momentum dependence is just determined through a simple delta function. This simple model has been frequently used for a leading order estimation of the B-meson decays. It is argued that when including the 3-particle Fock states' contribution, the transverse momentum distribution may be expanded to a certain degree other than such a simple delta function. The contributions from the higher Fock states' may not be too small, e.g. in Ref.[6], the 3-particle contributions are estimated by attaching an extra gluon to the internal off-shell quark line, and then $(1/m_b)$ power suppression is readily induced. Recently, a simple model for the B-meson wave function up to next-to-leading Fock state has been raised in Ref.[7], where relations between the 2- and 3- particle wavefunctions derived from the QCD equations of motion and the heavy quark symmetry [8], especially two constraints derived from the gauge field equation of motion, are employed. More explicitly, the normalized B-meson wave functions in the compact parameter b-space can be written as [7]

$$\Psi_+(\omega, b) = \frac{\omega}{\omega_0^2} \exp\left(-\frac{\omega}{\omega_0}\right) \left(\Gamma[\delta] J_{\delta-1}[\kappa] + (1-\delta)\Gamma[2-\delta] J_{1-\delta}[\kappa]\right) \left(\frac{\kappa}{2}\right)^{1-\delta} \tag{1}$$

and

$$\Psi_-(\omega, b) = \frac{1}{\omega_0} \exp\left(-\frac{\omega}{\omega_0}\right) \left(\Gamma[\delta] J_{\delta-1}[\kappa] + (1-\delta)\Gamma[2-\delta] J_{1-\delta}[\kappa]\right) \left(\frac{\kappa}{2}\right)^{1-\delta}, \tag{2}$$

with $\omega_0 = 2\bar{\Lambda}/3$, $\kappa = \sqrt{\omega(2\bar{\Lambda} - \omega)}b$ and δ is in the range of $(0, 1)$. In the above model, only two typical phenomenological parameters $\bar{\Lambda}$ and δ are introduced. $\bar{\Lambda}$ stands for the effective mass of B meson that determines the B-meson's leading Fock state behavior, while δ is a typical parameter that determines the broadness of the B-meson transverse distribution, and

the uncertainty caused by δ is of order $\mathcal{O}(1/m_b^2)$. This solution provides a practical framework for constructing the B-meson LC WFs and hence is meaningful for phenomenological applications.

The $B \to \pi$ and $B \to K$ transition form factors $F^{B\to\pi}_{+,0,T}$ and $F^{B\to K}_{+,0,T}$ provide a good platform to determine the possible regions for $\bar{\Lambda}$ and δ. In the large recoil energy region, the $B \to \pi$, K transition form factors can be studied both under the modified pQCD factorization approach (or the so-called k_T factorization approach) [9, 10], the QCD sum rule [11] and the later developed QCD light-cone sum rule (LCSR) [12]. The properties of the involving light pseudo-scalar wave functions can be more precisely determined within the QCD LCSR analysis or the pQCD calculations from the more sensitive processes like pionic/kaonic electromagnetic form factors to compare with the experimental data [13, 14, 15] or from the lattice calculation [16], so we shall directly take them to be the ones favored in the literature, and then the main uncertainties for the present k_T factorization approach come from the B-meson wave function. In fact, by varying the undetermined parameters of the pionic/kaonic wave functions within reasonable regions determined in literature [1], it can be found that the main uncertainty of the $B \to \pi$ and $B \to K$ transition form factors really comes from that of the B-meson wave function [2]. On the other hand, within the QCD LCSR approach with proper correlator, it has been found that the main uncertainties in estimation of form factors come from the pionic and kaonic twist-2 and twist-3 wave functions. A systematic QCD LCSR calculation of $B \to \pi$, K transition form factors has been finished in Refs.[19, 20, 21, 22] by including the one-loop radiative corrections to the pionic/kaonic twist-2 and twist-3 contributions. So through a comparative study of the form factors with the k_T factorization approach and the QCD LCSR, one can derive the reasonable regions for the two undetermined parameters $\bar{\Lambda}$ and δ of the B-meson wave function, which is the main purpose of the present letter.

The $B \to \pi$ and $B \to K$ transition form factors $F^{B\to\pi}_{+,0,T}$ and $F^{B\to K}_{+,0,T}$ are defined as follows:

$$\langle P(p)|V_\mu^P|B(p_B)\rangle = \left[(p_B+p)_\mu - \frac{M_B^2-M_P^2}{q^2}q_\mu\right]F_+^{B\to K}(q^2) + \frac{M_B^2-M_P^2}{q^2}q_\mu F_0^{B\to P}(q^2) \quad (3)$$

[1] In literature, some attempts to derive the properties of the pionic wave function from $B \to \pi$ within the LCSR approach can be found in Refs.[17, 18].

[2] Such a discussion for the $B \to K$ vector and scalar form factors can be found in Ref.[31].

and

$$\langle P(p)|J_\mu^{P,\sigma}|B(p_B)\rangle = i\frac{F_T^{B\to P}(q^2)}{M_B+M_P}\left[q^2(p_B+p)_\mu-(M_B^2-M_P^2)q_\mu\right], \tag{4}$$

where P stands for the pseudo-scalar meson π or K respectively, the momentum transfer $q=p_B-p$, the vector currents $V_\mu^\pi=\bar{u}\gamma_\mu b$ and $V_\mu^K=\bar{s}\gamma_\mu b$, the tensor currents $J_\mu^{\pi,\sigma}=q^\mu\bar{d}\sigma_{\mu\nu}b$ and $J_\mu^{K,\sigma}=q^\mu\bar{s}\sigma_{\mu\nu}b$.

Within the k_T factorization approach, the $B\to P$ transition form factors are dominated by a single gluon exchange in the lowest order. Following the same procedure as described in Ref.[23], we obtain all the mentioned transition form factors in the transverse configuration $\mathbf{b}$-space up to order $\mathcal{O}(1/m_b^2)$, i.e.

$$\begin{aligned}
F_+^{B\to P}(q^2) &= \frac{\pi C_F}{N_c}f_Pf_BM_B^2\int d\xi dx\int b_Bdb_B\ b_Pdb_P\ \alpha_s(t)\times\exp(-S(x,\xi,b_P,b_B;t))\\
&\times S_t(x)S_t(\xi)\Bigg\{\Bigg[\Psi_P(x,b_P)\Big((x\eta+1)\Psi_B(\xi,b_B)-\bar\Psi_B(\xi,b_B)\Big)\\
&+\frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot\left((1-2x)\Psi_B(\xi,b_B)+\left(x+\frac{1}{\eta}-1\right)\bar\Psi_B(\xi,b_B)\right)\\
&-\frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_P)}{6}\cdot\left(\left(1+2x-\frac{2}{\eta}\right)\Psi_B(\xi,b_B)-(1+x-\frac{1}{\eta})\bar\Psi_B(\xi,b_B)\right)\\
&+\frac{m_0^p}{M_B}\Psi_\sigma(x,b_P)\left(\Psi_B(\xi,b_B)-\frac{\bar\Psi_B(\xi,b_B)}{2}\right)\Bigg]h_1(x,\xi,b_P,b_B)\\
&-(1+\eta+x\eta)\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}[M_B\Delta(\xi,b_B)]h_2(x,\xi,b_P,b_B)\\
&+\Bigg[\Psi_P(x,b_p)\left(-\xi\bar\eta\Psi_B(\xi,b_B)+\frac{\Delta(\xi,b_B)}{M_B}\right)+2\frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot\\
&\left((1-\xi)\Psi_B(\xi,b_B)+\xi(1-\frac{1}{\eta})\bar\Psi_B(\xi,b_B)+2\frac{\Delta(\xi,b_B)}{M_B}\right)\Bigg]h_1(\xi,x,b_B,b_P)\Bigg\},
\end{aligned}\tag{5}$$

$$\begin{aligned}
F_0^{B\to P}(q^2) &= \frac{\pi C_F}{N_c}f_Pf_BM_B^2\int d\xi dx\int b_Bdb_B\ b_Pdb_P\ \alpha_s(t)\times\exp(-S(x,\xi,b_P,b_B;t))\\
&\times S_t(x)S_t(\xi)\Bigg\{\Bigg[\Psi_P(x,b_P)\eta\Big((x\eta+1)\Psi_B(\xi,b_B)-\bar\Psi_B(\xi,b_B)\Big)\\
&+\frac{m_0^p}{M_B}\Psi_p(x,b_P)((2-\eta-2x\eta)\Psi_B(\xi,b_B)-(1-\eta-x\eta)\bar\Psi_B(\xi,b_B))\\
&-\frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_P)}{6}\cdot(\eta(2x-1)\Psi_B(\xi,b_B)-(1+x\eta-\eta)\bar\Psi_B(\xi,b_B))\\
&+\eta\frac{m_0^p}{M_B}\Psi_\sigma(x,b_P)\left(\Psi_B(\xi,b_B)-\frac{\bar\Psi_B(\xi,b_B)}{2}\right)\Bigg]h_1(x,\xi,b_P,b_B)\\
&-[3-\eta-x\eta]\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}[M_B\Delta(\xi,b_B)]h_2(x,\xi,b_P,b_B)
\end{aligned}$$

$$
\begin{aligned}
&+ \left[\Psi_P(x,b_P)\eta\left(\xi\bar{\eta}\Psi_B(\xi,b_B)+\frac{\Delta(\xi,b_B)}{M_B}\right)\right.\\
&+ 2\frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot\Big((\eta(1+\xi)-2\xi)\Psi_B(\xi,b_B)-(\eta\xi-\xi)\bar{\Psi}_B(\xi,b_B)\\
&\left.+ 2(2-\eta)\frac{\Delta(\xi,b_B)}{M_B}\Big)\right]h_1(\xi,x,b_B,b_P)\Big\}
\end{aligned}
\qquad (6)
$$

and

$$
\begin{aligned}
F_T^{B\to P}(q^2) \;=\; & \frac{\pi C_F}{N_c}f_P f_B M_B^2\int d\xi dx\int b_B db_B\; b_P db_P\;\alpha_s(t)\times\exp[-S(x,\xi,b_P,b_B;t)]\\
&\times S_t(x)S_t(\xi)\Big\{\Big[\Psi_P(x,b_P)\left(\Psi_B(\xi,b_B)-\bar{\Psi}_B(\xi,b_B)\right)+\frac{m_0^p}{M_B}\Psi_p(x,b_P)\cdot\\
&\left(\frac{1}{\eta}\bar{\Psi}_B(\xi,b_B)-x\Psi_B(\xi,b_B)\right)+\frac{m_0^p}{M_B}\frac{\Psi'_\sigma(x,b_P)}{6}\left(\frac{x\eta+2}{\eta}\Psi_B(\xi,b_B)\right.\\
&\left.-\frac{1}{\eta}\bar{\Psi}_B(\xi,b_B)\right)+\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}\Psi_B(\xi,b_B)\Big]h_1(x,\xi,b_P,b_B)-\frac{m_0^p}{M_B}\frac{\Psi_\sigma(x,b_P)}{6}\\
&[M_B\Delta(\xi,b_P)]h_2(x,\xi,b_P,b_B)+\Big[\Psi_P(x,b_P)\left(\frac{\Delta(\xi,b_B)}{M_B}-\xi\Psi_B(\xi,b_B)\right)+\\
&2\frac{m_0^p}{M_B}\Psi_p(x,b_P)\left(\Psi_B(\xi,b_B)-\frac{\xi}{\eta}\bar{\Psi}_B(\xi,b_B)\right)\Big]h_1(\xi,x,b_B,b_P)\Big\},
\end{aligned}
\qquad (7)
$$

where the integration over the azimuth angles have been implicitly done, the transverse momentum dependence for both the hard scattering part and the non-perturbative wave functions, the Sudakov effects and the threshold effects are included to give a consistent analysis of the form factors up to $\mathcal{O}(1/m_b^2)$. And the two introduced functions

$$
\begin{aligned}
h_1(x,\xi,b_P,b_B) \;=\; & K_0(\sqrt{\xi x\eta}\;M_B b_B)\Big[\theta(b_B-b_P)I_0(\sqrt{x\eta}\;M_B b_P)K_0(\sqrt{x\eta}\;M_B b_B)\\
&+\theta(b_P-b_B)I_0(\sqrt{x\eta}\;M_B b_B)K_0(\sqrt{x\eta}\;M_B b_P)\Big]
\end{aligned}
\qquad (8)
$$

and

$$
\begin{aligned}
h_2(x,\xi,b_P,b_B) \;=\; & \frac{b_B}{2\sqrt{\xi x\eta}M_B}K_1(\sqrt{\xi x\eta}\;M_B b_B)\Big[\theta(b_B-b_P)I_0(\sqrt{x\eta}\;M_B b_P)K_0(\sqrt{x\eta}\;M_B b_B)\\
&+\theta(b_P-b_B)I_0(\sqrt{x\eta}\;M_B b_B)K_0(\sqrt{x\eta}\;M_B b_P)\Big],
\end{aligned}
\qquad (9)
$$

where the functions I_i (K_i) are the modified Bessel functions of the first (second) kind with the *i-th* order. Implicitly, we have set

$$
\Psi_B=\Psi_B^+\;,\quad \bar{\Psi}_B=\Psi_B^+-\Psi_B^-\;,\quad \Delta(\xi,b_B)=M_B\int_0^\xi d\xi'[\Psi_B^-(\xi',b_B)-\Psi_B^+(\xi',b_B)], \qquad (10)
$$

where B-meson wave functions Ψ_B^+ and Ψ_B^- are taken as Eqs.(1,2) that include 3-particle Fock states' contributions. It can be found that the contributions from $\bar{\Psi}_B$ is rightly power suppressed to that of Ψ_B. Such a definition for Ψ_B and $\bar{\Psi}_B$ is often adopted in the literature to simplify the calculation, since the the contribution from $\bar{\Psi}_B$ can be safely neglected for the leading order estimation [3]. The factor $\exp(-S(x,\xi,b_P,b_B;t))$ contains the Sudakov logarithmic corrections and the renormalization group evolution effects of both the wave functions and the hard scattering amplitude,

$$S(x,\xi,b_P,b_B;t) = \left[s(x,b_P,M_b) + s(\bar{x},b_P,M_b) + s(\xi,b_B,M_b) - \frac{1}{\beta_1}\ln\frac{\hat{t}}{\hat{b}_\pi} - \frac{1}{\beta_1}\ln\frac{\hat{t}}{\hat{b}_B}\right], \quad (11)$$

where $\hat{t} = \ln(t/\Lambda_{QCD})$, $\hat{b}_B = \ln(1/b_B\Lambda_{QCD})$, $\hat{b}_\pi = \ln(1/b_P\Lambda_{QCD})$ and $s(x,b,Q)$ is the Sudakov exponent factor, whose explicit form up to next-to-leading log approximation can be found in Ref.[24]. $S_t(x)$ and $S_t(\xi)$ come from the threshold resummation effects and here we take a simple parametrization proposed in Refs.[25, 26],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c\,, \quad (12)$$

where the parameter c is determined around 0.3 for the present case. The hard scale t in $\alpha_s(t)$ and the Sudakov form factor might be varied for the different hard scattering parts and here we need two t_i, which can be chosen as the largest scale of the virtualities of internal particles [25, 27],

$$t_1 = \mathrm{MAX}\left(\sqrt{x\eta}M_B, 1/b_P, 1/b_B\right),\ t_2 = \mathrm{MAX}\left(\sqrt{\xi\eta}M_B, 1/b_P, 1/b_B\right). \quad (13)$$

The Fourier transformation for the transverse part of the wave function is defined as

$$\Psi(x,\mathbf{b}) = \int_{|\mathbf{k}|<1/b} d^2\mathbf{k}_\perp \exp\left(-i\mathbf{k}_\perp\cdot\mathbf{b}\right)\Psi(x,\mathbf{k}_\perp), \quad (14)$$

where Ψ stands for Ψ_P, Ψ_p, Ψ_σ, Ψ_B, $\bar{\Psi}_B$ and Δ, respectively. The upper edge of the integration $|\mathbf{k}_\perp| < 1/b$ is necessary to ensure that the wave function is soft enough [30]. And we take the phenomenological parameter, which is a scale characterized by the chiral perturbation theory, $m_0^p \simeq 1.30$ GeV for pion [32] and $m_0^p \simeq 1.70 GeV$ for kaon [27] respectively. For the twist-3 $\Psi_p(x,\mathbf{k}_\perp)$ and $\Psi_\sigma(x,\mathbf{k}_\perp)$, we take them to be the ones constructed in Ref.[23]

[3] Another definition is also adopted in literature, $\Psi_B = (\Psi_B^+ + \Psi_B^-)/2$ and $\bar{\Psi}_B = (\Psi_B^+ - \Psi_B^-)/2$, however in this definition Ψ_B and $\bar{\Psi}_B$ should be treated on the equal footing as pointed out in Ref.[23].

for the pionic case and Ref.[31] for the kaonic case respectively. As for the twist-2 pion and kaon WFs, they can be constructed based on their first two Gegenbauer moments and the BHL prescription [28], i.e.

$$\Psi_\pi(x,\mathbf{k}_\perp) = [1 + B_\pi C_2^{3/2}(2x-1) + C_\pi C_4^{3/2}(2x-1)]\frac{A_\pi}{x(1-x)}\exp\left[-\beta_\pi^2\left(\frac{\mathbf{k}_\perp^2+m_q^2}{x(1-x)}\right)\right], \quad (15)$$

and

$$\Psi_K(x,\mathbf{k}_\perp) = [1+B_K C_1^{3/2}(2x-1)+C_K C_2^{3/2}(2x-1)]\frac{A_K}{x(1-x)}\exp\left[-\beta_K^2\left(\frac{\mathbf{k}_\perp^2+m_q^2}{x}+\frac{\mathbf{k}_\perp^2+m_s^2}{1-x}\right)\right], \quad (16)$$

where $q = u,\ d$, $C_{1,2}^{3/2}(1-2x)$ are Gegenbauer polynomials. The constitute quark masses are set to be: $m_q = 0.30\mathrm{GeV}$ and $m_s = 0.45\mathrm{GeV}$. The four undetermined parameters can be determined by the first two Gegenbauer moments a_2^π and a_4^π (or a_1^K and a_2^K), the normalization condition and the constraint $\langle\mathbf{k}_\perp^2\rangle_K^{1/2} \approx \langle\mathbf{k}_\perp^2\rangle_\pi^{1/2} = 0.350\mathrm{GeV}$ [29], where the average value of the transverse momentum square is defined as

$$\langle\mathbf{k}_\perp^2\rangle_{\pi,K}^{1/2} = \frac{\int dx d^2\mathbf{k}_\perp|\mathbf{k}_\perp^2||\Psi_{\pi,K}(x,\mathbf{k}_\perp)|^2}{\int dx d^2\mathbf{k}_\perp|\Psi_{\pi,K}(x,\mathbf{k}_\perp)|^2}.$$

As a comparison, within the QCD LCSR approach with proper choosing correlator, it has been found that the main uncertainties in estimation of those form factors come from the pionic and kaonic twist-2 and twist-3 wave functions, especially from the twist-3 wave function Ψ_p whose distribution amplitude (DA) ϕ_p has the asymptotic behavior $\phi_p(x)|_{q^2\to\infty} \to 1$ for both the pionic and kaonic cases. Two typical ways have been adopted to suppress the uncertainty caused by the twist-3 WFs. One way is raised by Ref.[33], i.e. an improved LCSR with proper chiral current was adopted to eliminate the contributions from the most uncertain pionic and kaonic twist-3 wave functions and to enhance the reliability of the LCSR calculations [34, 35, 36]. The other way is to do a systematic QCD LCSR calculation of $B \to \pi$, K transition form factors by including the one-loop radiative corrections to both the pionic/kaonic twist-2 contributions and the twist-3 contributions [19, 20, 21]. A comparison of these two approaches to improve the LCSR estimation has been done in Ref.[35], which shows that these two treatments are equivalent to each other, at least for $F_+^{B\to\pi}(q^2)$ and $F_+^{B\to K}(q^2)$. Here we shall adopt the LCSR results of Ref.[19] to do our discussion, where $F_{+,0,T}^{B\to\pi}$ and $F_{+,0,T}^{B\to K}$ have been parameterized in the following form [19]

$$F_{+,0,T}^{B\to P}(q^2) = f^{as}(q^2) + a_1^P(\mu_0)f^{a_1^P}(q^2) + a_2^P(\mu_0)f^{a_2^P}(q^2) + a_4^P(\mu_0)f^{a_4^P}(q^2), \quad (17)$$

where P stands for π or K, f^{as} contains the contributions to the form factor from the asymptotic DA and all higher-twist effects from three-particle quark-quark-gluon matrix elements, $f^{a_1^P,a_2^P,a_4^P}$ contains the contribution from the higher Gegenbauer term of DA that is proportional to a_1^P, a_2^P and a_4^P respectively. μ_0 is the factorization scale which separates long-distance physics (distribution amplitudes) from the short-distance physics (hard-scattering amplitudes). The explicit expressions of f^{as,a_1^P,a_2^P,a_4^P} can be found in Table V and Table IX of Ref.[19]. Since the form factors have been split into contributions from different Gegenbauer moments, and the uncertainties other than the Gegenbauer moment itself have been absorbed into the uncertainty of the functions f^{as} and $f^{a_1^P,a_2^P,a_4^P}$, then one can conveniently obtain the LCSRs with various Gegenbauer moments with the help of Eq.(17). Eq.(17) allows one to use the possible newly developed pionic or kaonic twist-2 DA Gegenbauer moments to do the discussion, and it is the reason why we chose the LCSRs of Ref.[19] to do our comparison.

In doing the numerical calculations, we take $\Lambda^{(n_f=4)}_{\overline{MS}} = 250MeV$ and

$$f_\pi = 130.7 \pm 0.1 \pm 0.36MeV, \quad f_K = 159.8 \pm 1.4 \pm 0.44MeV, \tag{18}$$

where the decay constants f_π and f_K are taken from Ref.[37]. As for f_B, it can be calculated from the QCD sum rules and the lattice QCD. Here we fix its value to be $190MeV$, i.e. the center value derived from the lattice QCD [38], which is consistent with the Belle experiment $f_B = 229^{+36+34}_{-31-37}MeV$ [39]. The change of f_B can influence the final results on the transition form factors a lot and a more precise f_B shall be helpful to improve the precision of our present estimation, e.g. it can be found that a variation of f_B by 10% shall bring less than 3% extra uncertainty to the allowable range of $\bar{\Lambda}$ and δ. A more detailed discussion of f_B with the LCSRs up to next-to-leading order shall be presented elsewhere [40].

The $B \to \pi$ transition form factors $F^{B\to\pi}_{+,0,T}$ are shown in Fig.(1), where a_2^π and a_4^π are within the region determined by the two suggested constraints [19]: $a_2^\pi(1\mathrm{GeV}) + a_4^\pi(1\mathrm{GeV}) = 0.1 \pm 0.1$ [41] and $-\frac{9}{4}a_2^\pi(1\mathrm{GeV}) + \frac{45}{16}a_4^\pi(1\mathrm{GeV}) + \frac{3}{2} = 1.2 \pm 0.3$ [42]. The results of the k_T factorization with $\bar{\Lambda} \in [0.50, 0.55]$ and $\delta \in [0.25, 0.30]$ are shown by a fuscous shaded band with $0 \le q^2 \le 10GeV^2$, and the LCSR results with its 12% uncertainty [19] are shown by a grey band with $0 \le q^2 \le 15GeV^2$. It can be found that the form factors $F^{B\to\pi}_{+,0,T}$ decrease with the increment of $\bar{\Lambda}$ and increase with the increment of δ. The upper edge of the fuscous shaded band is for $\bar{\Lambda} = 0.50GeV$ and $\delta = 0.30$, and the lower edge of the fuscous shaded band is for $\bar{\Lambda} = 0.55GeV$ and $\delta = 0.25$. One may observe that k_T factorization results of

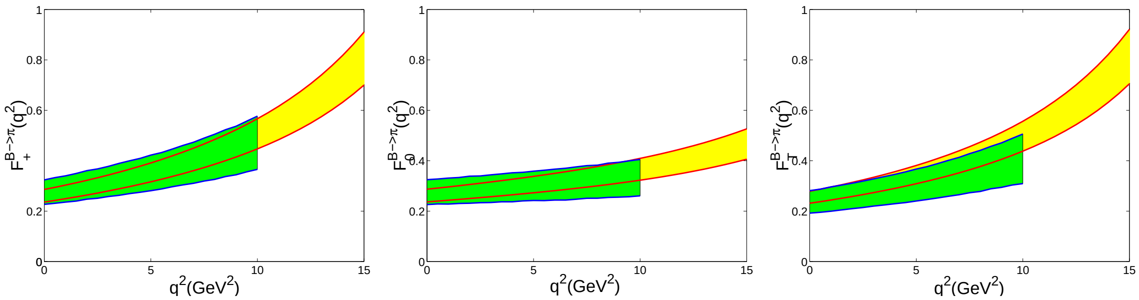

FIG. 1: Comparative results of $F^{B\to\pi}_{+,0,T}(q^2)$ within the k_T factorization approach and the QCD LCSR, where the fuscous shaded band stands for k_T factorization results and the grey band stands for the QCD LCSR results [19]. The upper edge of the fuscous shaded band is for $\bar\Lambda = 0.50GeV$ and $\delta = 0.30$, and the lower edge of the fuscous shaded band is for $\bar\Lambda = 0.55GeV$ and $\delta = 0.25$.

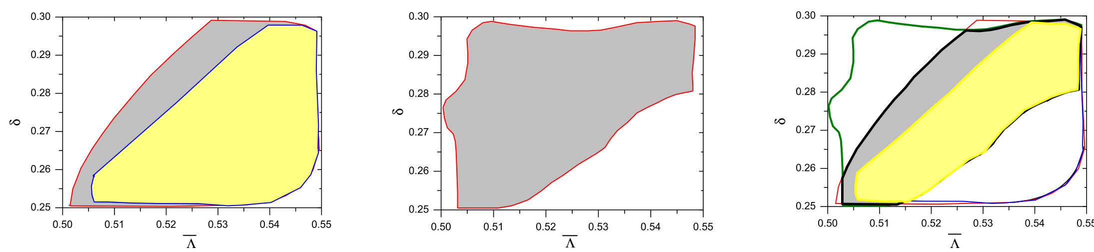

FIG. 2: Allowable regions for $\bar\Lambda$ and δ from $B \to \pi$ form factors $F^{B\to\pi}_{+,0,T}(0)$, which are shown by fuscous shaded bands respectively. Left diagram stands for the constraints from $F^{B\to\pi}_{+,0}(0)$, Middle one is for $F^{B\to\pi}_{T}(0)$ and Right one is the combined results of the two, where the fainter band is for $F^{B\to\pi}_{+}(0) = 0.26 \pm 0.02$ [43].

$F^{B\to\pi}_{+,0,T}$ can agree with that of the QCD LCSR at small value of q^2 with proper values for $\bar\Lambda$ and δ. And the best fit of all the three form factors within these two approaches are obtained for $\bar\Lambda \simeq 0.525GeV$ and $\delta \simeq 0.275$.

At $q^2 = 0$, the QCD LCSR gives [19]: $F^{B\to\pi}_{+,0}(0) = 0.258 \pm 0.031$ and $F^{B\to\pi}_{T}(0) = 0.253 \pm 0.028$. If requiring the k_T factorization results for $F^{B\to\pi}_{+,0,T}(0)$ to be consistent with that of LCSR, one can obtain the possible ranges for $\bar\Lambda$ and δ, i.e. $\bar\Lambda \in [0.50, 0.55]GeV$ and $\delta \in [0.25, 0.30]$, and furthermore, $\bar\Lambda$ and δ should be correlated in a way as shown in Fig.(2). In Fig.(2), the left diagram stands for the constraints from $F^{B\to\pi}_{+,0}(0)$, the middle one is for $F^{B\to\pi}_{T}(0)$ and the right one is the combined results from $F^{B\to\pi}_{+,0}(0)$ and $F^{B\to\pi}_{T}(0)$. Recently, a nearly model-independent analysis for $F^{B\to\pi}_{+}(q^2)$ based on the BaBar experimental data

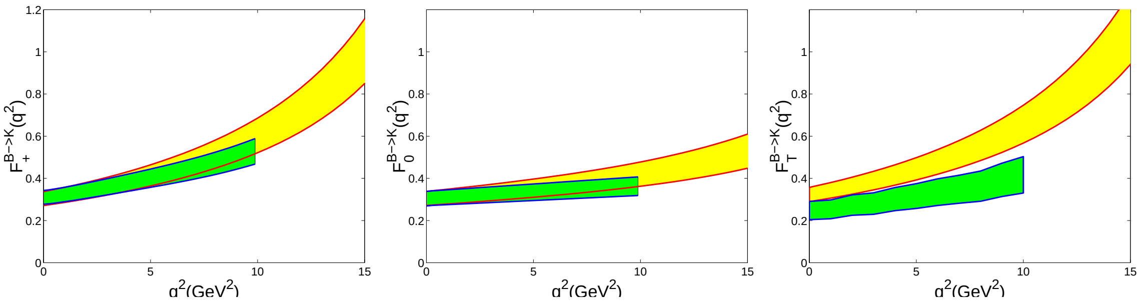

FIG. 3: Comparative results of $F^{B\to K}_{+,0,T}(q^2)$ within the k_T factorization approach and the QCD LCSR, where the fuscous shaded band stands for k_T factorization results and the fainter band stands for the QCD LCSR results [19]. The upper edge of the fuscous shaded band is for $\bar{\Lambda} = 0.50GeV$ and $\delta = 0.30$, and the lower edge of the fuscous shaded band is for $\bar{\Lambda} = 0.55GeV$ and $\delta = 0.25$.

on $B \to \pi l\nu$ has been given in Ref.[43], which shows $F_+^{B\to\pi}(0) = 0.26 \pm 0.02$. If requiring the k_T factorization results for $F_+^{B\to\pi}(0)$ to be within this smaller region, we can obtain a more stringent constraints for $\bar{\Lambda}$ and δ as is shown by the fainter band of Fig.(2). Note, all the contours are obtained by sampling 10,000 points for $F^{B\to\pi}_{+,0,T}(0)$ to be within the allowable region respectively.

The $B \to K$ transition form factors $F^{B\to K}_{+,0,T}$ are shown in Fig.(3), where the results of the k_T factorization with $\bar{\Lambda} \in [0.50, 0.55]$ and $\delta \in [0.25, 0.30]$ are shown by a fuscous shaded band with $0 \leq q^2 \leq 10GeV^2$ and the results of the LCSR with its $(12\% + 3\%)$ uncertainty [19] are shown by a fainter band with $0 \leq q^2 \leq 15GeV^2$, where the extra 3% error is introduced due to the $a_1^K(1GeV)$ uncertainty. The first Gegenbauer moment a_1^K stands for the $SU_f(3)$-breaking of the $B \to K$ transition form factors in comparison to the $B \to \pi$ form factors, and it has been studied by the light-front quark model [44], the QCD sum rule [45, 46, 47, 48] and the lattice calculation [49, 50] and etc. In Ref.[45], the QCD sum rule for the diagonal correlation function of local and nonlocal axial-vector currents is used, in which the contributions of condensates up to dimension six and the $\mathcal{O}(\alpha_s)$-corrections to the quark-condensate term are taken into account. The moments derived there are close to that of the lattice calculation [49, 50], so we shall take $a_1^K(1\text{GeV}) = 0.05 \pm 0.02$ and $a_2^K(1\text{GeV}) = 0.10 \pm 0.05$ to do our estimation. Further more a discussion of the uncertainty of a_1^K to the k_T factorization approach can be found in Ref.[31], which shows that such kind of uncertainty is quite small in comparison to the uncertainties caused by the change of $\bar{\Lambda}$

and δ.

Similar to the $B \to \pi$ case, it can be found that the form factors $F^{B\to K}_{+,0,T}$ decrease with the increment of $\bar{\Lambda}$ and increase with the increment of δ. The upper edge of the fuscous shaded band is for $\bar{\Lambda} = 0.50GeV$ and $\delta = 0.30$, and the lower edge of the fuscous shaded band is for $\bar{\Lambda} = 0.55GeV$ and $\delta = 0.25$. One may observe that k_T factorization results of $F^{B\to K}_{+,0}(q^2)$ can agree with that of the QCD LCSR at small value of q^2 with proper values for $\bar{\Lambda}$ and δ. However the value of $F^{B\to K}_{T}(q^2)$ is lower than that of QCD LCSR almost in the whole q^2 region, so no proper ranges for $\bar{\Lambda}$ and δ can be derived from the comparison of $F^{B\to K}_{T}(q^2)$. More explicitly, at the largest recoil region $q^2 = 0$, the QCD LCSR gives [19]: $F^{B\to K}_{+,0}(0) = 0.331 \pm 0.041 + 0.25(a^K_1(1GeV) - 0.17)$ and $F^{B\to K}_{T}(0) = 0.358 \pm 0.037 + 0.25(a^K_1(1GeV) - 0.17)$, which shows that $F^{B\to K}_{+,0}$ is **smaller** than $F^{B\to K}_{T}(0)$ under the same parameters. While the k_T factorization approach gives: $F^{B\to K}_{+,0}(0) = 0.30 \pm 0.04$ and $F^{B\to K}_{T}(0) = 0.25 \pm 0.03$ for $\bar{\Lambda} \in [0.50, 0.55]GeV$ and $\delta \in [0.25, 0.30]$, which shows that $F^{B\to K}_{+,0}$ is **bigger** than $F^{B\to K}_{T}(0)$ under the same parameters. This difference between the $B \to \pi$ and $B \to K$ maybe explained by the $SU_f(3)$-breaking symmetry effect. It can be found that within the k_T factorization and Ref.[51], a similar $SU_f(3)$-breaking effects can be found for all the three form factors $F^{B\to K}_{+,0,T}(0)$, more explicitly, $[F^{B\to K}_{+,0,T}(0)/F^{B\to\pi}_{+,0,T}(0)] \sim 1.08$ for the k_T factorization approach and $[F^{B\to K}_{+,0,T}(0)/F^{B\to\pi}_{+,0,T}(0)] \sim 1.24$ for Ref.[51]. While Ref.[19] gives somewhat different $SU_f(3)$-breaking effects: $[F^{B\to K}_{+,0}(0)/F^{B\to\pi}_{+,0}(0)] \sim 1.16$ and $[F^{B\to K}_{T}(0)/F^{B\to\pi}_{T}(0)] \sim 1.28$. In the literature, new QCD sum rules for $B \to \pi$, K form factors have been derived from the correlation functions expanded near the light cone in terms of B-meson distribution, which are consistent with the present k_T factorization results and show that $F^{B\to\pi}_{+,0}(0) = 0.25 \pm 0.05$, $F^{B\to\pi}_{T}(0) = 0.21 \pm 0.04$, $F^{B\to K}_{+,0}(0) = 0.31 \pm 0.04$ and $F^{B\to K}_{T}(0) = 0.27 \pm 0.04$ [51]. Very recently, another independent LCSR calculation on the $B \to \pi$ and $B \to K$ transition form factors have been presented in Refs.[20, 21], which gives $F^{B\to\pi}_{+,0}(0) = 0.26^{+0.04}_{-0.03}$, $F^{B\to\pi}_{T}(0) = 0.255 \pm 0.035$, $F^{B\to K}_{+,0}(0) = 0.36^{+0.05}_{-0.04}$ and $F^{B\to K}_{T}(0) = 0.38 \pm 0.05$. Such results of $B \to \pi$ is very close to our present PQCD results, while the results of $B \to K$ form factors $F^{B\to K}_{+,0}(0)$ and $F^{B\to K}_{T}(0)$ are larger that ours, and hence a larger $SU_f(3)$-breaking effect $[F^{B\to K}_{T}(0)/F^{B\to\pi}_{T}(0)] \sim 1.49$ [21] is presented there. Such a discrepancy is mainly caused by a larger value $a^K_1(1GeV) = 0.10 \pm 0.04$, the treatment of the b-quark mass and also some other parameters like f_B and etc.. So a comparative study of the form factors and a precise QCD LCSR calculation on the form factor with tensor current, including the

$SU_f(3)$-breaking effect and all its possible uncertainties, is necessary to clarify the present situation [22]. So we shall only make a comparison of $F^{B\to K}_{+,0}(0)$ to decide possible range for $\bar{\Lambda}$ and δ at the present. And by sampling 10,000 points for $F^{B\to K}_{+,0}(0)$ to be within the region derived from QCD LCSR [19], it can be found that all the points in $\bar{\Lambda} \in [0.50, 0.55] GeV$ and $\delta \in [0.25, 0.30]$ are allowable.

In the present paper, the properties of the B-meson light-cone wave function up to next-to-leading order Fock state expansion have been studied through a comparative study of the $B \to \pi$ and $B \to K$ transition form factors under both the k_T factorization approach and the QCD LCSR approach. The QCD LCSR approach with proper correlator shall have no relation to the B-meson DA but shall be quite sensitive to the light mesons' DAs, while the k_T factorization approach is insensitive to the light mesons' distribution amplitudes but depends on the B-meson DA heavily, so these two approaches are compensated to each other. A more precise QCD LCSR results shall be helpful to obtain a more accurate information on the B-meson wave function, and vice versa.

We have applied the k_T factorization approach to do a systematical study on the $B \to \pi$ and $B \to K$ transition form factors up to $\mathcal{O}(1/m_b^2)$, where the transverse momentum dependence for the wave function, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to acquire a more reasonable result. By comparing with the QCD LCSR results, it has been found that when the two typical phenomenological parameters $\bar{\Lambda} \in [0.50, 0.55]$ and $\delta \in [0.25, 0.30]$ (the correlation relation between $\bar{\Lambda}$ and δ can be found in the Right diagram of Fig.(2)), which control the leading and next-to-leading Fock states' contributions respectively, the results of $F^{B\to\pi}_{+,0,T}(q^2)$ and $F^{B\to K}_{+,0,T}(q^2)$ from these two approaches are consistent with each other in the large recoil energy region. Inversely, one can derive the reasonable regions for the two undetermined parameters of the simple B-meson model wave function as shown in Eqs.(1, 2). The slight discrepancy ($\sim 20\%$) of $F^{B\to\pi,K}_T(q^2)$ between the k_T factorization approach and the QCD LCSR results of Ref.[19] may be compensated by carefully taking the $SU_f(3)$-breaking effects into the QCD sum rule calculation. As a byproduct, it can be found that the $SU_f(3)$-breaking effects are small in the $B \to K$ transition form factors. Finally, one can adopt the B-meson wave functions up to next-to-leading order Fock state expansion to present a more precise studies on the B-meson decays up to $\mathcal{O}(1/m_b^2)$. It is noted that at the present, only the main uncertain sources are considered to determine the properties of the B-meson wave function, a more

precise study that includes a more precise pseudo-scalar wave functions shall be helpful to improve our understanding on the B-meson wave function.

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